

# Discoveries

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# Randomness rules

## Cheaters beware: Benford's Law says there's a pattern to the distribution of digits in lists

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What people don't know about the math of random numbers could get them in trouble with the law.

Cheaters who aren't familiar with a mathematical rule about the way digits are distributed may be more likely to get caught cooking the books.

The rule, known as Benford's Law, states that the digits that make up the numbers in a list will be distributed so that about 30 percent of the numbers will start with the digit 1, 18 percent will begin with 2, and so on until a mere 4.6 percent of the numbers have a 9 in the first slot.

The digits in everything from daily sales figures at Albertson's to plankton counts off Nova Scotia follow Benford's Law, said Ted Hill, a mathematician from the Georgia Institute of Technology in

### MATHEMATICS

Atlanta.

What the cheaters don't know is that every time the books get cooked, the pattern of digits will change so that the data don't fit the law, said Mark Nigrini, an accounting professor at the Cox School of Business at Southern Methodist University. Dr. Nigrini conducts training seminars for the Institute of Internal Auditors to teach auditors how to recognize fraud or error in lists of numbers.

Even though Benford's Law was discovered more than a century ago, mathematicians have had the tools to show how it operates only for a few years, said Dr. Hill.

Benford's Law was first described over a century ago by astronomer Simon Newcomb. Newcomb noticed that the pages of logarithm books were dirtier in the

front sections of the books — where numbers starting with one are listed — than the last few pages — where numbers beginning with nine are found. Newcomb noted the strange phenomenon in an 1881 letter to the *American Journal of Mathematics*, but it went unnoticed until Frank Benford, a physicist from the General Electric Co., rediscovered the dirty pages trend.

Benford presented 20,229 observations in a 1938 paper in the *Proceedings of the American Philosophical Society* to show how ubiquitous the law is. Benford and Newcomb both derived the same formula to describe the law, but neither was able to offer any explanation of why it should work.

What Benford did was pull numbers from sources that really had no relationship to each other. He mixed football scores with population records with

lengths of rivers to arrive at his rule. "This is something we're taught not to do from childhood on," Dr. Hill said. "You don't mix apples and oranges."

But combining these unrelated numbers gave Benford a "distribution of distributions" — a law of true randomness — that is universal, said Dr. Hill.

Benford's Law is so unexpected that most people don't believe it when they first hear about it, Dr. Hill said. "People have a subconscious idea of what they think is random," he said, and that idea doesn't mesh very well with reality.

People just don't expect a pattern of any sort to emerge from a list of random numbers, Dr. Hill said. "If I asked you to give me a list of random numbers, you're probably not going to say three, three, three very often," he said. But a run of threes should appear as often in a list of

Please see BENFORD'S on Page 8D.

# Benford's Law finds order in seemingly random lists

Continued from Page 7D.

random numbers as the series three, one, seven.

This psychological bias against patterns in chance is the Achilles' heel that Dr. Nigrini is teaching the auditors to look for when analyzing a company's books. Dr. Nigrini developed computer programs that analyze how well sets of numbers fit the expected distribution of digits prescribed by Benford's Law.

It wasn't always easy to convince people that Benford's Law was anything more than a mathematical curiosity, let alone that it could have practical applications, said Dr. Nigrini. "I was very lonely in the beginning. It was only Hill and I talking about it," he said. "I spent a lot of time proving Benford. Does it work? Is it true?"

Not only is Benford's Law true, says Dr. Hill, but it works even if one

were to switch to another numbering system. Here on Earth, math is usually done in a system based on the number 10. Martians with seven fingers instead of 10 might have a mathematical system based on seven. If the Martians were to try to find a law that would govern the distribution of digits in the seven-based system, they too would come up with Benford's Law, Dr. Hill said.

The other feature of Benford's Law that makes it particularly useful for accountants is that it works regardless of the scale of the numbers, Dr. Nigrini said. Benford's Law will apply even if you convert dollars to yen or pounds or pesos.

Benford's Law is counterintuitive, said Dr. Nigrini. It would seem that all the digits should be equally represented, but consider the Dow Jones index. If the Dow starts at 1,000, it



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must increase 100 percent to reach 2000. A 50 percent increase is needed to raise the Dow from 2000 to 3000. But by the time the index reaches 9000, it only takes an 11 percent jump to bring it to 10,000. It is much faster to zip through the larger digits than the smaller digits.

Even though mathematicians have a better understanding of how Benford's Law works, there are still many mysteries surrounding it, said Dr. Hill. Nobody knows how fast you

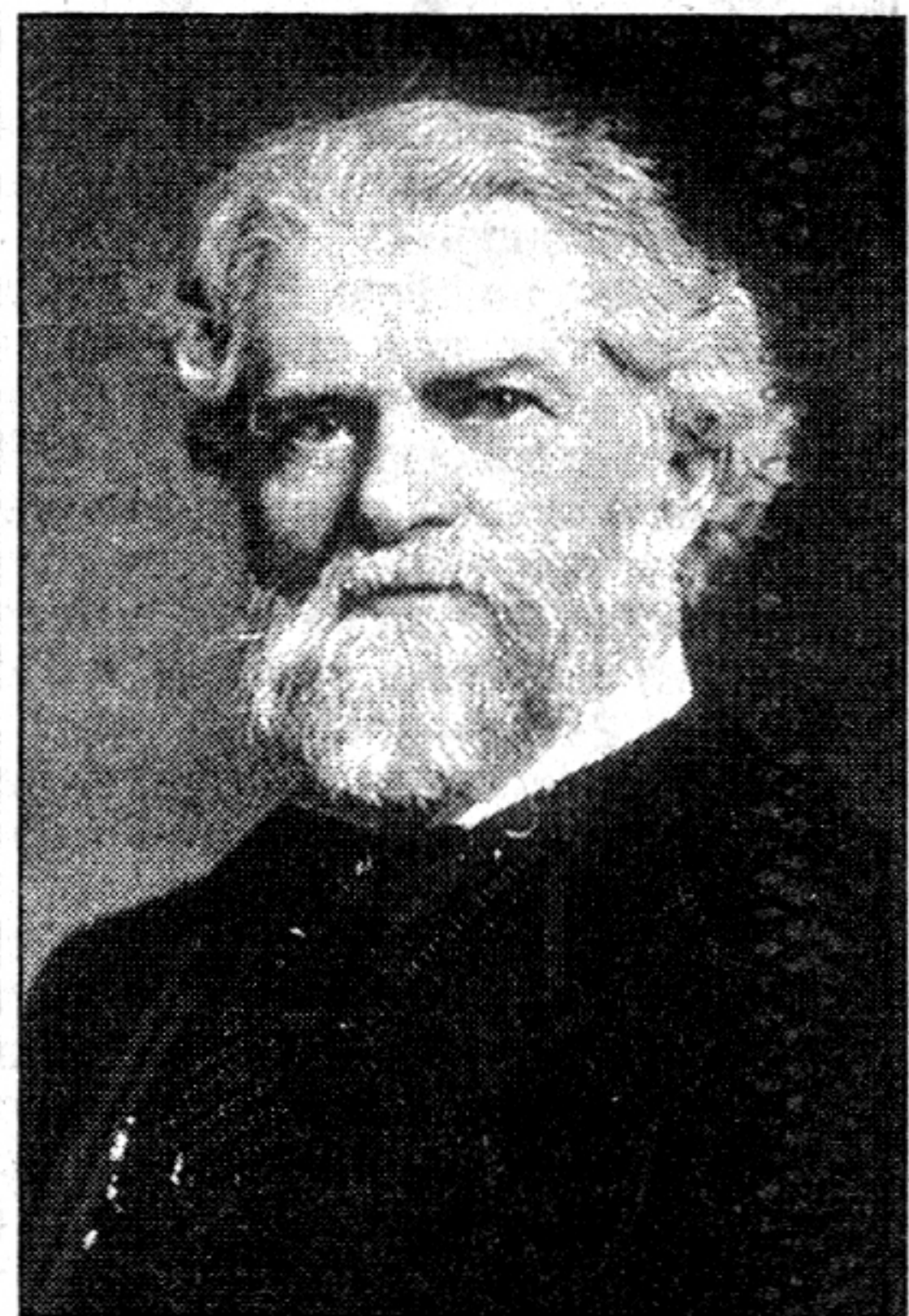
can get to Benford's Law — how many numbers need to be in a list before the digits will show a Benford distribution.

Not every list of numbers will follow Benford's Law, Dr. Nigrini said. The telephone book is a list of assigned numbers that won't follow the law. Neither will a list of the individual prices of items in the supermarket — there is an artificial limit placed on prices. However, those individual prices are combined

in a total at the check-out counter, and a sample of the daily check-out receipts from a store should comply with Benford's Law.

Benford's Law won't help anyone win the lottery — every number has an equal chance of being chosen and doesn't fit the rule — but it could help run businesses more efficiently, said Dr. Nigrini. In addition to ferreting out fraud, Dr. Nigrini's computer programs pick out blips in company records that might indicate inefficient business practices or accounting mistakes like adding in the same invoice more than once.

Any time a duplication of numbers occurs, the distribution of digits can diverge from Benford's Law. Picking out these anomalies is not only Dr. Nigrini's life's work, it is how he wants to be remembered. "I want inscribed on my tombstone, 'He found abnormal duplications.'"



Astronomical Society of the Pacific

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